

Coordinated Output Regulation of Heterogeneous Linear Systems under Switching Topologies[★]

Ziyang Meng^a, Tao Yang^a, Dimos V. Dimarogonas^a, Karl H. Johansson^a

^aACCESS Linnaeus Centre, School of Electrical Engineering, Royal Institute of Technology, Stockholm 10044, Sweden.

Abstract

This paper constructs a framework to describe and study the coordinated output regulation problem for multiple heterogeneous linear systems. Each agent is modeled as a general linear multiple-input multiple-output system with an autonomous exosystem which represents the individual offset from the group reference for the agent. The multi-agent system as a whole has a group exogenous state which represents the tracking reference for the whole group. Under the constraints that the group exogenous output is only locally available to each agent and that the agents have only access to their neighbors' information, we propose observer-based feedback controllers to solve the coordinated output regulation problem using output feedback information. A high-gain approach is used and the information interactions are allowed to be switched over a finite set of fixed networks containing both graphs that have a directed spanning tree and graphs that do not. The fundamental relationship between the information interactions, the dwell time, the non-identical dynamics of different agents, and the high-gain parameters is given. Simulations are shown to validate the theoretical results.

Key words: Heterogeneous linear dynamic systems; Coordinated output regulation; Switching communication topology

1 Introduction

Coordinated control of multi-agent systems has recently drawn large attention due to its broad applications in physical, biological, social, and mechanical systems [2–5]. The key idea of “coordination” algorithm is to realize a global emergence using only local information interactions [6, 7]. The coordination problem of a single-integrator network is fully studied with an emphasis on the system robustness to the input time delays and switching communication topologies [6–9], discrete-time dynamical models [10, 11], nonlinear couplings [12], the convergence speed evaluation [13], the effects of quantization [14], and the leader-follower tracking [15].

Following these ideas, the study of coordination of multiple linear dynamic systems becomes an attractive and fruitful research direction for the control community recently. For example, the authors of [16] generalize the existing works on coordination of multiple single-integrator systems to the case of multiple linear time-invariant single-input systems. For a

network of neutrally stable systems and polynomially unstable systems, the author of [17] proposes a design scheme for achieving synchronization. The case of switching communication topologies is considered in [18] and a so-called consensus-based observer is proposed to guarantee leaderless synchronization of multiple identical linear dynamic systems under a jointly connected communication topology. Similar problems are also considered in [19] and [20], where a frequently connected communication topology is studied in [19] and an assumption on the neutral stability is imposed in [20]. The authors of [21] propose a neighbor-based observer to solve the synchronization problem for general linear time-invariant systems. An individual-based observer and a low-gain technique are used in [22] to synchronize a group of linear systems with open-loop poles at most polynomially unstable. In addition, the classical Laplacian matrix is generalized in [23] to a so-called interaction matrix. A D-scaling approach is then used to stabilize this interaction matrix under both fixed and switching communication topologies. Synchronization of multiple heterogeneous linear systems has been investigated under both fixed and switching communication topologies [24–26]. A similar problem is studied in [27, 28], where a high-gain approach is proposed to dominate the non-identical dynamics of the agents. The cases of frequently connected and jointly connected communication topologies are studied in [29] and [30], respectively, where a slow switching condition and a fast switching condition are presented. Recently, the generalizations of

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Email addresses: ziyangm@kth.se (Ziyang Meng), taoyang@kth.se (Tao Yang), dimos@kth.se (Dimos V. Dimarogonas), kallej@kth.se (Karl H. Johansson).

coordination of multiple linear dynamic systems to the cooperative output regulation problem are studied in [31–33]. In addition, the study on the synchronization of homogeneous or heterogeneous networks with nonlinear couplings also attracts extensive attention [34–37].

In this paper, we generalize the classical output regulation problem of an individual linear dynamic system to the coordinated output regulation problem of multiple heterogeneous linear dynamic systems. We consider the case where each agent has an individual offset and simultaneously there is a group tracking reference. The individual offset and the group reference are generated by autonomous systems (*i.e.*, systems without inputs). Each individual offset is available to its corresponding agent while the group reference can be obtained only through constrained communication among the agents, *i.e.*, the group reference trajectory is available to only a subset of the agents. Our goal is to find an observer-based feedback controller for each agent such that the output of each agent converges to a given trajectory determined by the combination of the individual offset and the group reference. Motivated by the approach proposed in [27], we propose a unified observer to solve the coordinated output regulation problem of multiple heterogeneous general linear dynamics, where the open-loop poles of the agents can be exponentially unstable and the dynamics are allowed to be different both with respect to dimensions and parameters. This relaxes the common assumption of identical dynamics [17, 18, 20, 21, 29] or open-loop poles at most polynomially unstable [18, 20, 26]. The main contribution of this work is that the information interaction is allowed to be switching from a graph set containing both a directed spanning tree set and a disconnected graph set for the case of heterogeneous linear systems. This extends the existing works on the case of fixed communication topologies [17, 21, 27, 31]. The high-gain technique is used and the relationships between the dwell time [38], the non-identical dynamics among different agents and the high-gain parameters are also given.

The remainder of the paper is organized as follows. In Section 2, we give some basic definitions on network model. In Section 3, we formulate the problem of coordinated output regulation of multiple heterogeneous linear systems. We then propose the state feedback control law with a unified observer design in Section 4. Two case studies are given in Section 5. Numerical studies are carried out in Section 6 to validate our designs of observer-based controllers and a brief concluding remark is drawn in Section 7.

2 Network Model

We use graph theory to model the communication topology among agents. A directed graph G consists of a pair (\mathbf{V}, \mathbf{E}) , where $\mathbf{V} = \{v_1, v_2, \dots, v_n\}$ is a finite, nonempty set of nodes and $\mathbf{E} \subseteq \mathbf{V} \times \mathbf{V}$ is a set of ordered pairs of nodes. An edge (v_i, v_j) denotes that node v_j can obtain information from node v_i . All neighbors of node v_i are denoted as $N_i := \{v_j | (v_j, v_i) \in \mathbf{E}\}$. For an edge (v_i, v_j) in a directed

graph, v_i is the parent node and v_j is the child node. A directed path in a directed graph is a sequence of edges of the form $(v_i, v_j), (v_j, v_k), \dots$. A directed tree is a directed graph, where every node has exactly one parent except for one node, called the root, which has no parent, and the root has a directed path to every other node. A directed graph has a directed spanning tree if there exists at least one node having a directed path to all other nodes.

For a leader-follower graph $\bar{G} := (\bar{\mathbf{V}}, \bar{\mathbf{E}})$, we have $\bar{\mathbf{V}} = \{v_0, v_1, \dots, v_n\}$, $\bar{\mathbf{E}} \subseteq \bar{\mathbf{V}} \times \bar{\mathbf{V}}$, where v_0 is the leader and v_1, v_2, \dots, v_n denote the followers. The leader-follower adjacency matrix $\bar{A} = [a_{ij}] \in \mathbb{R}^{(n+1) \times (n+1)}$ is defined such that a_{ij} is positive if $(v_j, v_i) \in \bar{\mathbf{E}}$ while $a_{ij} = 0$ otherwise. Here we assume that $a_{ii} = 0$, $i = 0, 1, \dots, n$, and the leader has no parent, *i.e.*, $a_{0j} = 0$, $j = 0, 1, \dots, n$. The leader-follower “grounded” Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{n \times n}$ associated with \bar{A} is defined as $l_{ii} = \sum_{j=0}^n a_{ij}$ and $l_{ij} = -a_{ij}$, where $i \neq j$.

In this paper, we assume that the leader-follower communication topology $\bar{G}_{\sigma(t)}$ is time-varying and switching from a finite set $\{\bar{G}_k\}_{k \in \Gamma}$, where $\Gamma = \{1, 2, \dots, \delta\}$ is an index set and $\delta \in \mathbb{N}$ indicates its cardinality. We impose the technical condition that $\bar{G}_{\sigma(t)}$ is right continuous, where $\sigma : [t_0, \infty) \rightarrow \Gamma$ is a piecewise constant function of time. That is to say, $\bar{G}_{\sigma(t)}$ remains constant for $t \in [t_\ell, t_{\ell+1})$, $\ell = 0, 1, \dots$ and switches at $t = t_\ell$, $\ell = 1, 2, \dots$. In addition, we assume that $\inf_\ell (t_{\ell+1} - t_\ell) \geq \tau_d > 0$, $\ell = 0, 1, \dots$, with $\lim_{\ell \rightarrow \infty} t_\ell = \infty$, where τ_d is a constant known as the dwell time [38].

Let the sets $\{\bar{A}_k\}_{k \in \Gamma}$ and $\{L_k\}_{k \in \Gamma}$ be the leader-follower adjacency matrices and leader-follower grounded Laplacian matrices associated with $\{\bar{G}_k\}_{k \in \Gamma}$, respectively. Consequently, the time-varying leader-follower adjacency matrix and time-varying leader-follower grounded Laplacian matrix are defined as $\bar{A}_{\sigma(t)} = [a_{ij}(t)]$ and $L_{\sigma(t)} = [l_{ij}(t)]$.

Other notation in this paper: $\lambda_{\min}(P)$ and $\lambda_{\max}(P)$ denote, respectively, the minimum and maximum eigenvalues of a real symmetric matrix P , P^T denotes the transpose of P , and I_n denotes the $n \times n$ identity matrix.

3 Problem Formulation

3.1 Agent Dynamics

Suppose that we have n agents modeled by the linear MIMO systems:

$$\dot{x}_i = A_i x_i + B_i u_i, \quad (1)$$

where $x_i \in \mathbb{R}^{n_i}$ is the agent state, $u_i \in \mathbb{R}^{m_i}$ is the control input, $A_i \in \mathbb{R}^{n_i \times n_i}$, and $B_i \in \mathbb{R}^{n_i \times m_i}$.

Also suppose that there is an individual autonomous exosystem for each $v_i \in \mathbf{V}$,

$$\dot{\omega}_i = S_i \omega_i, \quad (2)$$

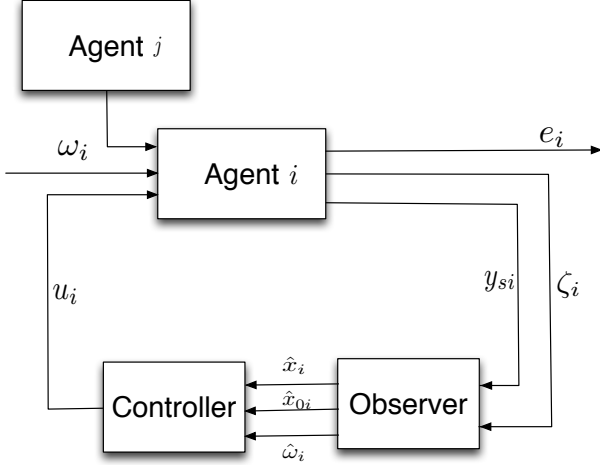


Fig. 1. Control architecture for agent v_i

where $\omega_i \in \mathbb{R}^{q_i}$ and $S_i \in \mathbb{R}^{q_i \times q_i}$.

In addition, there is a group autonomous exosystem for the multi-agent system as a whole:

$$\dot{x}_0 = A_0 x_0, \quad (3)$$

where $x_0 \in \mathbb{R}^{n_0}$ and $A_0 \in \mathbb{R}^{n_0 \times n_0}$.

3.2 Control Architecture

The control of each agent is supposed to have the structure shown in Fig. 1. More specifically, for the individual autonomous exosystem tracking, available output information for agent $v_i \in \mathbf{V}$ is

$$y_{si} = C_{si} x_i + C_{wi} \omega_i,$$

where $C_{si} \in \mathbb{R}^{p_1 \times n_i}$, and $C_{wi} \in \mathbb{R}^{p_1 \times q_i}$.

For the group autonomous exosystem tracking, only neighbor-based output information is available due to the constrained communication. This means that not all the agents have access to y_0 . The available information is the neighbor-based sum of each agent's own output relative to that of its' neighbors, i.e.,

$$\zeta_i = \sum_{j=0}^n a_{ij}(t)(y_{di} - y_{dj})$$

is available for each agent $v_i \in \mathbf{V}$, where $a_{ij}(t)$, $i = 0, 1, \dots, n$, $j = 0, 1, \dots, n$, is entry (i, j) of the adjacency matrix $\bar{A}_{\sigma(t)}$ associated with $\bar{G}_{\sigma(t)}$ defined in Section 2 at time t , y_{di} can be represented by $y_{di} = C_{di} x_i$, $i = 1, 2, \dots, n$ and $y_{d0} = C_0 x_0$, where $C_{di} \in \mathbb{R}^{p_2 \times n_i}$, $i = 1, 2, \dots, n$ and $C_0 \in \mathbb{R}^{p_2 \times n_0}$. Also, the relative estimation information is available using the same

communication topologies, i.e.,

$$\hat{\zeta}_i = \sum_{j=0}^n a_{ij}(t)(\hat{y}_i - \hat{y}_j)$$

is available for each agent $v_i \in \mathbf{V}$, where \hat{y}_i is an estimation produced internally by each agent $v_i \in \mathbf{V}$.

Fig. 2 gives an example of information flow among the agents and the group autonomous exosystem v_0 for $n = 3$ agents.

3.3 Switching Topologies

For the communication topology set $\{\bar{G}_k\}_{k \in \Gamma}$, we assume that \bar{G}_k , $\forall k \in \Gamma_c$ is a graph containing a directed spanning tree with v_0 rooted. Without loss of generality, we relabel $\Gamma_c := \{1, 2, \dots, \delta_1\}$ ($1 \leq \delta_1 \leq \delta$), where $\delta_1 \in \mathbb{N}$. The remaining graphs are labeled as \bar{G}_k , $\forall k \in \Gamma_d$, where $\Gamma_d := \{\delta_1 + 1, \delta_1 + 2, \dots, \delta\}$. Denote the graph set $\bar{\mathbb{G}}_c = \{\bar{G}_k\}_{k \in \Gamma_c}$ and the graph set $\bar{\mathbb{G}}_d = \{\bar{G}_k\}_{k \in \Gamma_d}$, respectively. We also denote $T_{t_0}^d(t)$ and $T_{t_0}^c(t)$ the total activation time when $\bar{G}_{\sigma(\varsigma)} \in \bar{\mathbb{G}}_d$ and total activation time when $\bar{G}_{\sigma(\varsigma)} \in \bar{\mathbb{G}}_c$ during $\varsigma \in [\bar{t}_0, t)$ for $\bar{t}_0 \geq t_0$.

Assumption 1 The dwell time τ_d is a positive constant.

Assumption 2 Given a positive constant κ , there exists a $\bar{t}_0 \geq t_0$ such that $T_{t_0}^c(t) \geq \kappa T_{t_0}^d(t)$ for all $t \geq \bar{t}_0$.

Remark 1 Note that a sufficient condition satisfying Assumption 2 is that $\bar{\mathbb{G}}_c$ is non-empty and given a $T > 0$ and $\tau_d > 0$, for any $t \geq t_0$, the switching signal $\sigma(t)$ satisfies $\{t | \bar{G}_{\sigma(t)} \in \bar{\mathbb{G}}_c\} \cap [t, t+T] \neq \emptyset$. Such a condition is also referred as “frequently connected” condition (i.e., the communication topology that contains a directed spanning tree is active frequently enough [19, 23]). Note that this condition implies that there exists a time sequence $0 = T_0 < T_1 < \dots < T_\ell \dots$ such that $\{t | \bar{G}_{\sigma(t)} \in \bar{\mathbb{G}}_c\} \cap [T_\ell, T_{\ell+1}] \neq \emptyset$, for all $\ell = 0, 1, \dots$, where $T_{\ell+1} - T_\ell \leq 2T$. Therefore, there exists a $\bar{t}_0 \in [t_0, t_0 + 2T]$ such that $T_{t_0}^c(t) \geq \frac{\tau_d}{2T} T_{t_0}^d(t)$ for all $t \geq \bar{t}_0$.

3.4 Control Objective

The control objective of each agent is to track a given trajectory determined by the combination of the group reference x_0 and the individual offset ω_i , $i = 1, 2, \dots, n$. Such a combination is captured by the coordinated output regulation tracking error (i.e., the total tracking error representing the combination of both individual tracking and group tracking of each agent):

$$e_i = D_{si} x_i + D_{wi} \omega_i + D_0 x_0. \quad (4)$$

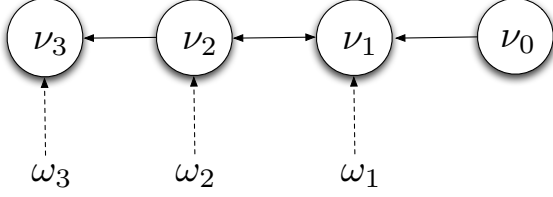


Fig. 2. Information flow associated with three agents ν_1, ν_2, ν_3 , the individual autonomous exosystems $\omega_1, \omega_2, \omega_3$, and the group autonomous exosystem ν_0

Thus, our objective is to guarantee that $\lim_{t \rightarrow \infty} e_i(t) = 0$. We design an observer-based controller with available individual output information and neighbor-based group output information to solve this problem.

For the system shown in Fig. 2, the overall control can correspond to a formation control problem, where ω_i encodes the relative position between each agent and the leader while the leader x_0 defines the overall motion of the group.

4 Coordinated Output Regulation with Unified Observer Design

As suggested by Fig. 1, the design procedure to solve the coordinated output regulation problem includes two main steps: the first one is the state feedback control design and the second one is the observer design for the group autonomous exosystem, the individual autonomous exosystem, and internal state information for each agent.

4.1 Redundant Modes

Before designing state feedback control and distributed observer, we need first to remove the redundant modes that have no effect on y_{si} and $y_{di} - y_{d0}$.

We impose the following assumptions on the structure of the systems.

Assumption 3

- $\left(A_i, \begin{bmatrix} C_{si} \\ C_{di} \end{bmatrix} \right)$, $i = 1, 2, \dots, n$ is observable.
- (S_i, C_{wi}) , $i = 1, 2, \dots, n$ is observable.
- (A_0, C_0) , $i = 1, 2, \dots, n$ is observable.

We first write the state and output of each agent in the compact form

$$\begin{bmatrix} \dot{x}_i \\ \dot{\omega}_i \\ \dot{x}_0 \end{bmatrix} = \begin{bmatrix} A_i & 0 & 0 \\ 0 & S_i & 0 \\ 0 & 0 & A_0 \end{bmatrix} \begin{bmatrix} x_i \\ \omega_i \\ x_0 \end{bmatrix} + \begin{bmatrix} B_i \\ 0 \\ 0 \end{bmatrix} u_i,$$

$$\begin{bmatrix} y_{si} \\ y_{di} - y_{d0} \end{bmatrix} = \begin{bmatrix} C_{si} & C_{wi} & 0 \\ C_{di} & 0 & -C_0 \end{bmatrix} \begin{bmatrix} x_i \\ \omega_i \\ x_0 \end{bmatrix}.$$

Given that Assumption 3 is satisfied, we can perform the state transformation given in Step 1 of [27] by considering ω_i

and x_0 together. We can construct a new state $\bar{x}_i = W_i \begin{bmatrix} x_i \\ \omega_i \\ x_0 \end{bmatrix}$

with the dynamics

$$\dot{\bar{x}}_i = \bar{A}_i \bar{x}_i + \bar{B}_i u_i = \begin{bmatrix} A_i & \bar{A}_{i12} \\ 0 & \bar{A}_{i22} \end{bmatrix} \bar{x}_i + \begin{bmatrix} B_i \\ 0 \end{bmatrix} u_i, \quad (6a)$$

$$\begin{bmatrix} y_{si} \\ e_{di} \end{bmatrix} = \bar{C}_i \bar{x}_i = \begin{bmatrix} C_{si} & \bar{C}_{i21} \\ C_{di} & \bar{C}_{i22} \end{bmatrix} \bar{x}_i. \quad (6b)$$

where $e_{di} = y_{di} - y_{d0}$, and the details designs on $W_i, \bar{A}_i, \bar{B}_i, \bar{C}_i$ are given in [27]. It was shown that pair (\bar{A}_i, \bar{C}_i) is observable and the eigenvalues of \bar{A}_{i22} are a subset of the eigenvalues of S_i and A_0 , $i = 1, 2, \dots, n$.

4.2 Regulated State feedback Control Law

We now design a controller to regulate e_i to zero for each agent based on the state information $\bar{x}_i = \begin{bmatrix} \bar{x}_{i1} \\ \bar{x}_{i2} \end{bmatrix}$, where $\bar{x}_{i1} \in \mathbb{R}^{n_i}$.

We impose the following assumptions on the structure of the systems.

Assumption 4

- (A_i, B_i) is stabilizable, $i = 1, \dots, n$.
- (A_i, B_i, D_{si}) is right-invertible, $i = 1, \dots, n$.
- (A_i, B_i, D_{si}) has no invariant zeros in the closed right-half complex plane that coincide with the eigenvalues of S_i or A_0 , $i = 1, \dots, n$.

Lemma 1 Let Assumption 4 hold. Then, the regulator equations (7) are solvable and the state-feedback controller $u_i = F_i(\bar{x}_{i1} - \Pi_i \bar{x}_{i2}) + \Gamma_i \bar{x}_{i2}$ ensures that $\lim_{t \rightarrow \infty} e_i(t) = 0$, $i = 1, 2, \dots, n$, where Π_i, Γ_i are the solutions of the following regulator equations

$$\Pi_i \bar{A}_{i22} = A_i \Pi_i + \bar{A}_{i12} + B_i \Gamma_i, \quad (7a)$$

$$0 = D_{si} \Pi_i + \begin{bmatrix} D_{wi} & D_0 \end{bmatrix}, \quad i = 1, 2, \dots, n, \quad (7b)$$

and F_i is chosen such that $A_i + B_i F_i$ is Hurwitz.

Proof: It follows from [39] and the similar analysis of proof of Lemma 3 in [27], we can show that the regulator equations

(7) are solvable given that Assumption 4 is satisfied. Then, by considering $\tilde{x}_{i2} = \bar{A}_{i22}\tilde{x}_{i2}$ as the exosystem and $\dot{x}_i = A_i x_i + B_i u_i$ as the system to be regulated for the classic output regulation result [40], we know that $u_i = F_i(\tilde{x}_{i1} - \Pi_i \tilde{x}_{i2}) + \Gamma_i \tilde{x}_{i2}$ ensures that $\lim_{t \rightarrow \infty} e_i(t) = 0$, $i = 1, 2, \dots, n$, where Π_i and Γ_i are the solutions of the regulator equations (7). ■

We next design observers to estimate \tilde{x}_i based on output information y_{si} and ζ_i for each agent.

4.3 Pseudo-identical Linear Transformation

Note that the individual offset ω_i can be estimated by y_{si} and the group reference x_0 can be estimated by $\hat{\zeta}_i$. In contrast, the internal state information x_i for each agent can be obtained by either y_{si} or $\hat{\zeta}_i$. In this section, we use the combination of y_{si} and $\hat{\zeta}_i$ to give a unified observer design.

We define $\chi_i = T_i \tilde{x}_i \in \mathbb{R}^{p\bar{n}}$, $i = 1, 2, \dots, n$, where $\bar{n} = n_0 + \max_{i=1,2,\dots,n}(n_i + q_i)$, $p = p_1 + p_2$, and

$$T_i = \begin{bmatrix} \bar{C}_i \\ \vdots \\ \bar{C}_i \bar{A}_i^{\bar{n}-1} \end{bmatrix}.$$

Note that T_i is full column rank since the pair (\bar{A}_i, \bar{C}_i) , $i = 1, 2, \dots, n$ is observable. This implies that $T_i^T T_i$ is non-singular. Therefore, it follows that

$$\dot{\chi}_i = (\mathcal{A} + \mathcal{L}_i)\chi_i + \mathcal{B}_i u_i, \quad (8a)$$

$$\begin{bmatrix} y_{si} \\ e_{di} \end{bmatrix} = \mathcal{C} \chi_i, \quad i = 1, 2, \dots, n, \quad (8b)$$

where $\mathcal{A} = \begin{bmatrix} 0 & I_{p(\bar{n}-1)} \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{p\bar{n} \times p\bar{n}}$, $\mathcal{L}_i = \begin{bmatrix} 0 \\ L_i \end{bmatrix}$, $\mathcal{B}_i = T_i \bar{B}_i$, $\mathcal{C} = \begin{bmatrix} I_p & 0 \end{bmatrix} \in \mathbb{R}^{p \times p\bar{n}}$ for some matrix $L_i \in \mathbb{R}^{p \times p\bar{n}}$.

4.4 Unified Observer Design

Motivated by [27], based on the available output information y_{si} and the neighbor-based group output information ζ_i , the distributed observer is proposed for (8) as

$$\begin{aligned} \dot{\hat{\chi}}_i &= (\mathcal{A} + \mathcal{L}_i)\hat{\chi}_i + \mathcal{B}_i u_i + S(\varepsilon) \mathcal{P} \mathcal{C}^T \\ &\times \left(\begin{bmatrix} y_{si} \\ \sum_{j=0}^n a_{ij}(t)(y_{di} - y_{dj}) \end{bmatrix} - \begin{bmatrix} \hat{y}_{si} \\ \sum_{j=0}^n a_{ij}(t)(\hat{y}_i - \hat{y}_j) \end{bmatrix} \right), \\ &i = 1, 2, \dots, n, \end{aligned} \quad (9)$$

where $a_{ij}(t)$, $i = 0, 1, \dots, n$, $j = 0, 1, \dots, n$, is entry (i, j) of the adjacency matrix $\bar{A}_{\sigma(t)}$ associated with $\bar{G}_{\sigma(t)}$ defined in

Section 2 at time t , $\hat{y}_{si} = \mathcal{C}_1 \hat{\chi}_i$, $\hat{y}_i = \mathcal{C}_2 \hat{\chi}_i$, $i = 1, \dots, n$, \mathcal{C}_1 is first p_1 rows of \mathcal{C} , \mathcal{C}_2 is the remaining p_2 rows of \mathcal{C} , and $\hat{y}_0 = 0$. In addition, $S(\varepsilon) = \text{diag}(I_p \varepsilon^{-1}, I_p \varepsilon^{-2}, \dots, I_p \varepsilon^{-\bar{n}})$, where $\varepsilon \in (0, 1]$ is a positive constant to be determined, and $\mathcal{P} = \mathcal{P}^T$ is a positive definite matrix satisfying

$$\mathcal{A} \mathcal{P} + \mathcal{P} \mathcal{A}^T - 2 \mathcal{P} \mathcal{C}^T \begin{bmatrix} I_{p_1} & 0 \\ 0 & \theta I_{p_2} \end{bmatrix} \mathcal{C} \mathcal{P} + I_{p\bar{n}} = 0, \quad (10)$$

where $\theta = \min_{k \in \Gamma_c} \beta_k$ and β_k will be determined later. Note that the existence of \mathcal{P} is due to the fact that $\left(\mathcal{A}, \begin{bmatrix} I_{p_1} & 0 \\ 0 & \sqrt{\theta} I_{p_2} \end{bmatrix} \mathcal{C} \right)$ is observable.

Lemma 2 • All the eigenvalues of L_k are in the closed right-half plane with those on the imaginary axis being simple, where L_k is associated with \bar{G}_k defined in Section 2, and some $\bar{G}_k \in \{\bar{G}_k\}_{k \in \Gamma}$.
• Furthermore, all the eigenvalues of L_k are in the open right-half plane for $\bar{G}_k \in \{\bar{G}_k\}_{k \in \Gamma_c}$.

Proof: See Theorem 4.29 in [41] and Lemma 1.6 in [42]. ■

Lemma 3 Let Assumptions 1, 2, 4, and 3 hold and assume that $\kappa \geq \frac{\alpha + 4 \max\{\theta, 1\} \lambda_{\max}^2(\mathcal{P})}{1 - \alpha}$, where $\alpha \in (0, 1)$. Then, there exists an $\varepsilon^* \in (0, 1]$ such that, if $\varepsilon \in (0, \varepsilon^*)$, $\lim_{t \rightarrow \infty} (\chi_i(t) - \hat{\chi}_i(t)) = 0$, $i = 1, 2, \dots, n$, for systems (9).

Proof: Note that for all $i = 1, 2, \dots, n$, $\sum_{j=0}^n a_{ij}(t)(y_{di} - y_{dj}) = \sum_{j=1}^n l_{ij}(t)(y_{dj} - y_{d0}) = \sum_{j=1}^n l_{ij}(t)e_{dj}$. Define $\tilde{\chi}_i = \chi_i - \hat{\chi}_i$. It then follows from (8) and (9) that

$$\dot{\tilde{\chi}}_i = (\mathcal{A} + \mathcal{L}_i)\tilde{\chi}_i - S(\varepsilon) \mathcal{P} \mathcal{C}^T \begin{pmatrix} y_{si} - \hat{y}_{si} \\ \sum_{j=1}^n l_{ij}(t)(e_{dj} - \hat{y}_j) \end{pmatrix}, \quad i = 1, 2, \dots, n,$$

where $l_{ij}(t)$, $i = 1, \dots, n$, $j = 1, \dots, n$, is the (i, j) th entry of the adjacency matrix $L_{\sigma(t)}$ associated with $\bar{G}_{\sigma(t)}$ defined in Section 2 at time t . It follows that

$$\dot{\tilde{\chi}}_i = (\mathcal{A} + \mathcal{L}_i)\tilde{\chi}_i - S(\varepsilon) \mathcal{P} \mathcal{C}^T \begin{pmatrix} \mathcal{C}_1 \tilde{\chi}_i \\ \mathcal{C}_2 \sum_{j=1}^n l_{ij}(t) \tilde{\chi}_j \end{pmatrix}, \quad i = 1, 2, \dots, n.$$

¹ The upper bound of the high-gain parameter may be conservative. We can use an empirical approach to derive a feasible ε^* in the practical applications.

By introducing $\xi_i = \varepsilon^{-1} S^{-1}(\varepsilon) \tilde{\chi}_i$ and after some manipulation, we have that

$$\varepsilon \dot{\xi}_i = (\mathcal{A} + \mathcal{L}_{i\varepsilon}) \xi_i - \mathcal{P} \mathcal{C}^T \left(\begin{bmatrix} \mathcal{C}_1 \xi_i \\ \mathcal{C}_2 \sum_{j=1}^n l_{ij}(t) \xi_j \end{bmatrix} \right), \quad i = 1, 2, \dots, n,$$

$$\text{where } \mathcal{L}_{i\varepsilon} = \begin{bmatrix} 0 \\ \varepsilon^{\bar{n}+1} L_i S(\varepsilon) \end{bmatrix} = O(\varepsilon).$$

Note that $\begin{bmatrix} \mathcal{C}_1 \xi_i \\ \mathcal{C}_2 \xi_i \end{bmatrix} = \mathcal{C} \xi_i$, for all $i = 1, 2, \dots, n$. The overall dynamics can be written as

$$\begin{aligned} \varepsilon \dot{\xi} &= (I_n \otimes \mathcal{A} + \mathcal{L}_\varepsilon - (I_n \otimes \mathcal{P} \mathcal{C}^T) \\ &\quad \times \left(I_n \otimes \begin{bmatrix} I_{p_1} & 0 \\ 0 & 0 \end{bmatrix} + L_\sigma \otimes \begin{bmatrix} 0 & 0 \\ 0 & I_{p_2} \end{bmatrix} \right) (I_n \otimes \mathcal{C})) \xi, \end{aligned} \quad (11)$$

where $\xi = [\xi_1^T, \xi_2^T, \dots, \xi_n^T]^T$ and $\mathcal{L}_\varepsilon = \text{diag}(\mathcal{L}_{1\varepsilon}, \mathcal{L}_{2\varepsilon}, \dots, \mathcal{L}_{n\varepsilon})$.

Note that $-L_k$, $k \in \Gamma_c$ is a Hurwitz stable matrix according to Lemma 2. Therefore, we can always guarantee that $-L_k + \beta_k I_n$ is also a Hurwitz stable matrix by choosing β_k sufficiently small. In particular, we choose β_k as a positive constant satisfying $\beta_k < \min \Re\{\lambda(L_k)\}$, $k \in \Gamma_c$, where $\min \Re\{\lambda(L_k)\}$ denote the minimum value of all the real parts of the eigenvalues of L_k . Then, we define piecewise Lyapunov function candidate $V_k = \varepsilon \xi^T (P_k \otimes \mathcal{P}^{-1}) \xi$, where P_k is positive definite matrix satisfying

$$P_k(-L_k + \beta_k I_n) + (-L_k + \beta_k I_n)^T P_k = -I_n < 0, \quad k \in \Gamma_c,$$

$$P_k(-L_k) + (-L_k)^T P_k \leq 0, \quad k \in \Gamma_d,$$

where the second inequality is due to Lemma 2.

It then follows that for all $k \in \Gamma_c$,

$$\begin{aligned} \dot{V}_k &\leq 2\xi^T (P_k \otimes \mathcal{P}^{-1} \mathcal{A}) \xi + 2\xi^T (P_k \otimes \mathcal{P}^{-1}) \mathcal{L}_\varepsilon \xi \\ &\quad - 2\xi^T \left(P_k \otimes \left(\mathcal{C}^T \begin{bmatrix} I_{p_1} & 0 \\ 0 & 0 \end{bmatrix} \mathcal{C} \right) \right) \xi \\ &\quad - 2\xi^T \left(P_k L_k \otimes \left(\mathcal{C}^T \begin{bmatrix} 0 & 0 \\ 0 & I_{p_2} \end{bmatrix} \mathcal{C} \right) \right) \xi \\ &\leq \xi^T \left(P_k \otimes \left(\mathcal{P}^{-1} \mathcal{A} + \mathcal{A}^T \mathcal{P}^{-1} - 2\theta \mathcal{C}^T \begin{bmatrix} 0 & 0 \\ 0 & I_{p_2} \end{bmatrix} \mathcal{C} \right. \right. \\ &\quad \left. \left. - 2\mathcal{C}^T \begin{bmatrix} I_{p_1} & 0 \\ 0 & 0 \end{bmatrix} \mathcal{C} \right) \right) \xi + 2\xi^T (P_k \otimes \mathcal{P}^{-1}) \mathcal{L}_\varepsilon \xi \end{aligned}$$

$$\begin{aligned} &- \xi^T \left((2P_k L_k - 2\theta P_k) \otimes (\mathcal{C}^T \begin{bmatrix} 0 & 0 \\ 0 & I_{p_2} \end{bmatrix} \mathcal{C}) \right) \xi \\ &\leq \xi^T (P_k \otimes (\mathcal{P}^{-1} (\mathcal{A} \mathcal{P} + \mathcal{P} \mathcal{A}^T \\ &\quad - 2\mathcal{P} \mathcal{C}^T \begin{bmatrix} I_{p_1} & 0 \\ 0 & \theta I_{p_2} \end{bmatrix} \mathcal{C} \mathcal{P}) \mathcal{P}^{-1})) \xi \\ &- \xi^T \left((P_k L_k + L_k^T P_k - 2\beta_k P_k) \otimes (\mathcal{C}^T \begin{bmatrix} 0 & 0 \\ 0 & I_{p_2} \end{bmatrix} \mathcal{C}) \right) \xi \\ &\quad + 2\lambda_{\max}(P_k) \lambda_{\max}(\mathcal{P}^{-1}) \|\mathcal{L}_\varepsilon\| \|\xi\|^2 \\ &\leq -\xi^T (P_k \otimes (\mathcal{P}^{-1} \mathcal{P}^{-1})) \xi \\ &\quad - \xi^T \left(I_n \otimes (\mathcal{C}^T \begin{bmatrix} 0 & 0 \\ 0 & I_{p_2} \end{bmatrix} \mathcal{C}) \right) \xi \\ &\quad + \frac{2\lambda_{\max}(P_k) \lambda_{\max}(\mathcal{P}^{-1}) \|\mathcal{L}_\varepsilon\|}{\varepsilon \lambda_{\min}(P_k) \lambda_{\min}(\mathcal{P}^{-1})} V_k \\ &\leq -\xi^T (P_k \otimes (\mathcal{P}^{-1} \mathcal{P}^{-1})) \xi, \\ &\quad + \frac{2\lambda_{\max}(P_k) \lambda_{\max}(\mathcal{P}^{-1}) \|\mathcal{L}_\varepsilon\|}{\varepsilon \lambda_{\min}(P_k) \lambda_{\min}(\mathcal{P}^{-1})} V_k \\ &\leq -\left(\frac{\lambda_{\min}(\mathcal{P}^{-1})}{\varepsilon} - \frac{2\lambda_{\max}(P_k) \lambda_{\max}(\mathcal{P}^{-1}) \|\mathcal{L}_\varepsilon\|}{\varepsilon \lambda_{\min}(P_k) \lambda_{\min}(\mathcal{P}^{-1})} \right) V_k, \end{aligned}$$

where we have used (10) and the fact that $\theta \leq \beta_k$, $k \in \Gamma_c$. It then follows that $\dot{V}_k \leq -\frac{1}{\varepsilon} \lambda_k V_k$, $\forall k \in \Gamma_c$, if $\|\mathcal{L}_\varepsilon\| < \frac{\lambda_{\min}(P_k) \lambda_{\min}(\mathcal{P})}{4\lambda_{\max}(P_k) \lambda_{\max}^2(\mathcal{P})}$, where $\lambda_k = \frac{1}{2\lambda_{\max}(\mathcal{P})}$, $\forall k \in \Gamma_c$.

On the other hand, for all $k \in \Gamma_d$, we have that

$$\begin{aligned} \dot{V}_k &\leq 2\xi^T (P_k \otimes (\mathcal{P}^{-1} \mathcal{A})) \xi + 2\xi^T (P_k \otimes \mathcal{P}^{-1}) \mathcal{L}_\varepsilon \xi \\ &\quad - 2\xi^T \left(P_k \otimes \left(\mathcal{C}^T \begin{bmatrix} I_{p_1} & 0 \\ 0 & 0 \end{bmatrix} \mathcal{C} \right) \right) \xi \\ &\quad - 2\xi^T \left(P_k L_k \otimes \left(\mathcal{C}^T \begin{bmatrix} 0 & 0 \\ 0 & I_{p_2} \end{bmatrix} \mathcal{C} \right) \right) \xi \\ &\leq \xi^T (P_k \otimes (\mathcal{P}^{-1} (\mathcal{A} \mathcal{P} + \mathcal{P} \mathcal{A}^T) \mathcal{P}^{-1})) \xi \\ &\quad + 2\lambda_{\max}(P_k) \lambda_{\max}(\mathcal{P}^{-1}) \|\mathcal{L}_\varepsilon\| \|\xi\|^2 \\ &\leq 2\xi^T \left(P_k \otimes \left(\mathcal{C}^T \begin{bmatrix} I_{p_1} & 0 \\ 0 & \theta I_{p_2} \end{bmatrix} \mathcal{C} \right) \right) \xi - \frac{\lambda_{\min}(\mathcal{P}^{-1})}{\varepsilon} V_k \\ &\quad + \frac{2\lambda_{\max}(P_k) \lambda_{\max}(\mathcal{P}^{-1}) \|\mathcal{L}_\varepsilon\|}{\varepsilon \lambda_{\min}(P_k) \lambda_{\min}(\mathcal{P}^{-1})} V_k, \end{aligned}$$

where we have used (10). Note that $\lambda_{\max} \left(\mathcal{C}^T \begin{bmatrix} I_{p_1} & 0 \\ 0 & \theta I_{p_2} \end{bmatrix} \mathcal{C} \right) = \max\{\theta, 1\}$. It follows that $\dot{V}_k \leq \frac{1}{\varepsilon} \lambda_k V_k$, $\forall k \in \Gamma_d$, if $\|\mathcal{L}_\varepsilon\| < \frac{\lambda_{\min}(P_k) \lambda_{\min}(\mathcal{P})}{2\lambda_{\max}(P_k) \lambda_{\max}^2(\mathcal{P})}$, where $\lambda_k = 2 \max\{\theta, 1\} \lambda_{\max}(\mathcal{P})$, $\forall k \in \Gamma_d$.

Following the similar analysis of [38, 43], we let $\sigma = p_j$ on $[t_{j-1}, t_j)$ for $p_j \in \Gamma$. Then, for any t satisfying $t_0 < t_1 < \dots < t_\ell < t < t_{\ell+1}$, define $V = \varepsilon \xi^T (P_{\sigma(t)} \otimes \mathcal{P}^{-1}) \xi$ for (11). We have that, $\forall \zeta \in [t_{j-1}, t_j)$,

$$\begin{aligned} V(\zeta) &\leq e^{-\frac{1}{\varepsilon} \lambda_{p_j}(\zeta - t_{j-1})} V(t_{j-1}) \\ &\leq e^{-\frac{1}{\varepsilon} \lambda^c(\zeta - t_{j-1})} V(t_{j-1}), \quad p_j \in \Gamma_c, \end{aligned}$$

$$\begin{aligned} V(\zeta) &\leq e^{\frac{1}{\varepsilon} \lambda_{p_j}(\zeta - t_{j-1})} V(t_{j-1}) \\ &\leq e^{\frac{1}{\varepsilon} \lambda^d(\zeta - t_{j-1})} V(t_{j-1}), \quad p_j \in \Gamma_d, \end{aligned}$$

where $\lambda^c = \min_{k \in \Gamma_c} \lambda_k = \frac{1}{2\lambda_{\max}(\mathcal{P})}$, $\lambda^d = \max_{k \in \Gamma_d} \lambda_k = 2\max\{\theta, 1\} \lambda_{\max}(\mathcal{P})$. Define $a = \frac{\lambda_{\max}(\mathcal{P})}{\lambda_{\min}(\mathcal{P})} \max_{k, j \in \Gamma} \frac{\lambda_{\max}(P_k)}{\lambda_{\min}(P_j)}$. We then know that $V(t_j) \leq a \lim_{t \uparrow t_j} V(t)$. Thus, it follows that

$$V(t) \leq a^\rho e^{\frac{1}{\varepsilon} \lambda^d T_0^d(t) - \frac{1}{\varepsilon} \lambda^c T_0^c(t)} V(\bar{t}_0),$$

where ρ denotes times of switching during $[\bar{t}_0, t)$. Note that $\rho \leq \frac{t - \bar{t}_0}{\tau_d}$. Given that $\kappa \geq \kappa^* = \frac{\lambda^d + \lambda}{\lambda^c - \lambda}$, for some $\lambda \in (0, \lambda^c)$, it follows from Assumption 2 that $T_{\bar{t}_0}^c(t) \geq \kappa^* T_{\bar{t}_0}^d(t)$ for all $t \geq \bar{t}_0$. This implies that $\lambda^d T_{\bar{t}_0}^d(t) - \lambda^c T_{\bar{t}_0}^c(t) \leq -\lambda(T_{\bar{t}_0}^d(t) + T_{\bar{t}_0}^c(t))$, for all $t \geq \bar{t}_0$ and we therefore know that

$$\begin{aligned} V(t) &\leq a^\rho e^{-\frac{1}{\varepsilon} \lambda(t - \bar{t}_0)} V(\bar{t}_0) \\ &\leq e^{\frac{t - \bar{t}_0}{\tau_d} \ln a - \frac{1}{\varepsilon} \lambda(t - \bar{t}_0)} V(\bar{t}_0) \\ &= e^{-\left(\frac{1}{\varepsilon} \lambda - \frac{\ln a}{\tau_d}\right)(t - \bar{t}_0)} V(\bar{t}_0). \end{aligned}$$

Furthermore, set $\lambda = \alpha \lambda^c$, where some $\alpha \in (0, 1)$. We then have that $\kappa^* = \frac{\alpha + 4\max\{\theta, 1\} \lambda_{\max}^2(\mathcal{P})}{1 - \alpha}$, and

$$V(t) \leq e^{-\left(\frac{\alpha}{2\varepsilon \lambda_{\max}(\mathcal{P})} - \frac{\ln a}{\tau_d}\right)(t - \bar{t}_0)} V(\bar{t}_0).$$

It follows that if $\varepsilon < \frac{\alpha \tau_d}{2\lambda_{\max}(\mathcal{P}) \ln a}$, we have for (11) that

$$\|\xi(t)\| \leq c^* e^{-\frac{1}{2} \left(\frac{\alpha}{2\varepsilon \lambda_{\max}(\mathcal{P})} - \frac{\ln a}{\tau_d}\right)(t - \bar{t}_0)} \|\xi(\bar{t}_0)\|,$$

where $c^* = \sqrt{\frac{\lambda_{\max}(\mathcal{P}) \max_{k \in \Gamma} \lambda_{\max}(P_k)}{\lambda_{\min}(\mathcal{P}) \min_{k \in \Gamma} \lambda_{\min}(P_k)}}$.

Therefore, we choose ε^* satisfying $\varepsilon^* < \frac{\alpha \tau_d}{2\lambda_{\max}(\mathcal{P}) \ln a}$ and $\|\mathcal{L}_{\varepsilon^*}\| < \min_{k \in \Gamma} \frac{\lambda_{\min}(P_k) \lambda_{\min}(\mathcal{P})}{4\lambda_{\max}(P_k) \lambda_{\max}^2(\mathcal{P})}$. It then follows that $\lim_{t \rightarrow \infty} (\chi_i(t) - \hat{\chi}_i(t)) = 0$, $i = 1, 2, \dots, n$. ■

Remark 2 Note that the condition of κ is necessary when the communication topology is switching. Roughly speaking, we need to guarantee that the influence of “the good

topology” beats that of “the bad topology” since the states of open-loop systems might diverge very fast due to the existence of unstable modes. The parameter κ is used to describe the relationship between $T_{\bar{t}_0}^c(t)$ and $T_{\bar{t}_0}^d(t)$, i.e., the remaining times of “good topology” and “bad topology”, respectively. The derived upper bound on κ might not be tight. However, we would like to emphasize that the significance is on the qualitative effects instead of quantitative effects. In practical applications, we can use an empirical approach to derive a feasible κ , as illustrated in Section 6.

From the unified observer design, we then have that

$$\hat{x}_i = (T_i^T T_i)^{-1} T_i^T \hat{\chi}_i = [\hat{x}_{i1}^T, \hat{x}_{i2}^T]^T, \quad i = 1, 2, \dots, n, \quad (14)$$

which will be used in the control input design.

4.5 Main Results

In this section, we show that the observer architecture introduced in the previous sections provide an asymptotically stable closed-loop system, as presented in Theorems 1 below. The observer-based controller is proposed as

$$u_i = F_i \hat{x}_{i1} + (\Gamma_i - F_i \Pi_i) \hat{x}_{i2}, \quad (15)$$

where Π_i and Γ_i are the solutions of the regulator equation (7), and \hat{x}_{i1} and \hat{x}_{i2} can be obtained from (9).

Theorem 1 Let Assumptions 1, 2, 3 and 4 hold and assume that $\kappa \geq \frac{\alpha + 4\max\{\theta, 1\} \lambda_{\max}^2(\mathcal{P})}{1 - \alpha}$, where $\alpha \in (0, 1)$, θ and \mathcal{P} are given by (10). Then, there exists $\varepsilon^* \in (0, 1]$ such that, if $\varepsilon \in (0, \varepsilon^*]$, (15) ensures that $\lim_{t \rightarrow \infty} e_i(t) = 0$, $i = 1, 2, \dots, n$, for the multi-agent system (1)-(4).

Proof: Follows from Lemmas 1 and 3, and the separation principle. ■

Remark 3 If the leader-follower communication topology \bar{G} is time-invariant, Assumptions 1 and 2 are not needed, and therefore the high-gain parameter only depends on the non-identical dynamics of the agents.

5 Case Studies

We notice that (9) give a unified way using y_{si} and ζ_i to estimate x_i , ω_i , and x_0 . One drawback of such a general approach is that the dimension of the observer $\hat{\chi}_i$ may be unnecessarily large for some cases with special structures. We next give particular structural designs on two special cases, i.e., the case when (A_i, C_{si}) is observable and the case when (A_i, C_{di}) is observable².

² These two cases are special cases of the first item of Assumption 3.

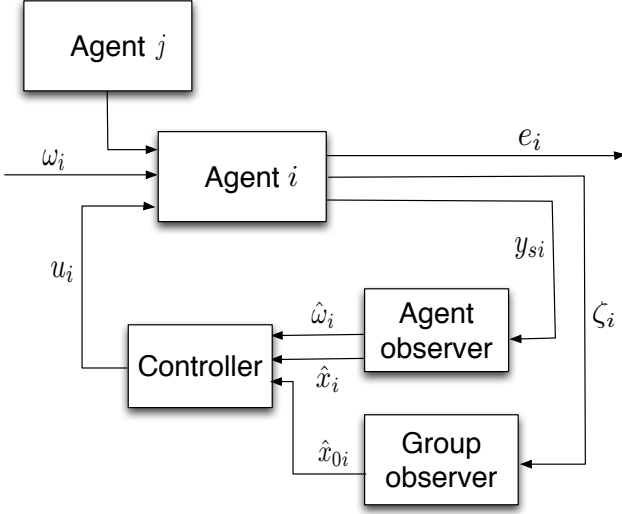


Fig. 3. Control architecture for agent v_i

5.1 Case I: (A_i, C_{si}) is observable

In this section, we use y_{si} to estimate both x_i and ω_i and use ζ_i to estimate x_0 . The control of each agent has the structure shown in Fig. 3.

We replace the first item of Assumption 3 with that (A_i, C_{si}) , for all $i = 1, 2, \dots, n$ is observable.

Step I: redundant mode remove

We first write the state and output of x_i and ω_i for each agent in the compact form

$$\begin{aligned} \begin{bmatrix} \dot{x}_i \\ \dot{\omega}_i \end{bmatrix} &= \begin{bmatrix} A_i & 0 \\ 0 & S_i \end{bmatrix} \begin{bmatrix} x_i \\ \omega_i \end{bmatrix} + \begin{bmatrix} B_i \\ 0 \end{bmatrix} u_i, \\ y_{si} &= \begin{bmatrix} C_{si} & C_{wi} \end{bmatrix} \begin{bmatrix} x_i \\ \omega_i \end{bmatrix}. \end{aligned}$$

We can then construct a new state $\bar{x}_i = W_i \begin{bmatrix} x_i \\ \omega_i \end{bmatrix}$ and perform the state transformation such that

$$\begin{aligned} \dot{\bar{x}}_i &= \bar{A}_i \bar{x}_i + \bar{B}_i u_i = \begin{bmatrix} A_i & \bar{A}_{i12} \\ 0 & \bar{A}_{i22} \end{bmatrix} \bar{x}_i + \begin{bmatrix} B_i \\ 0 \end{bmatrix} u_i, \\ y_{si} &= \bar{C}_i \bar{x}_i = \begin{bmatrix} C_{si} & \bar{C}_{i12} \end{bmatrix} \bar{x}_i. \end{aligned}$$

Similar to Section 4.1, we can show that the pair (\bar{A}_i, \bar{C}_i) is observable and the eigenvalues of \bar{A}_{i22} are a subset of the eigenvalues of S_i , $i = 1, 2, \dots, n$.

Step II: agent observer

Based on the information of the individual output information y_{si} , the following individual observer for each agent v_i is proposed

$$\dot{\hat{x}}_i = \bar{A}_i \hat{x}_i + \bar{B}_i u_i + K_{ai} (\bar{C}_i \hat{x}_i - y_{si}), \quad (18a)$$

$$[\hat{x}_i^T, \hat{\omega}_i^T]^T = W_i^{-1} \hat{x}_i, \quad i = 1, 2, \dots, n, \quad (18b)$$

where K_{ai} is chosen such that $\bar{A}_i + K_{ai} \bar{C}_i$ is Hurwitz stable, $i = 1, 2, \dots, n$.

Step III: group observer

We transform (3) into its canonical form. Define $\chi_0 = T_0 x_0 \in \mathbb{R}^{pn_0}$, where

$$T_0 = \begin{bmatrix} C_0 \\ \vdots \\ C_0 A_0^{n_0-1} \end{bmatrix}.$$

Therefore, it follows that

$$\begin{aligned} \dot{\chi}_0 &= (\mathcal{A}_0 + \mathcal{L}_0) \chi_0, \\ y_0 &= \mathcal{C}_0 \chi_0, \end{aligned}$$

$$\begin{aligned} \text{where } \mathcal{A}_0 &= \begin{bmatrix} 0 & I_{p(n_0-1)} \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{pn_0 \times pn_0}, \mathcal{L}_0 = \\ &= \begin{bmatrix} 0 \\ C_0 A_0^{n_0} (T_0^T T_0)^{-1} T_0^T \end{bmatrix}, \mathcal{C}_0 = \begin{bmatrix} I_p & 0 \end{bmatrix} \in \mathbb{R}^{p \times pn_0}. \end{aligned}$$

Then, based on the neighbor-based group output information ζ_i , the distributed observer is proposed

$$\begin{aligned} \dot{\hat{\chi}}_{0i} &= (\mathcal{A}_0 + \mathcal{L}_0) \hat{\chi}_{0i} - S(\epsilon) \mathcal{P} \mathcal{C}_0^T \left(\sum_{j=0}^n a_{ij}(t) (y_{di} - y_{dj}) \right. \\ &\quad \left. - \sum_{j=0}^n a_{ij}(t) (\hat{y}_i - \hat{y}_j) \right), \end{aligned} \quad (20a)$$

$$\hat{x}_{0i} = (T_0^T T_0)^{-1} T_0^T \hat{\chi}_{0i}, \quad i = 1, 2, \dots, n, \quad (20b)$$

where $a_{ij}(t)$, $i = 0, 1, \dots, n$, $j = 0, 1, \dots, n$, is entry (i, j) of the adjacency matrix $\bar{A}_{\sigma(t)}$ associated with $\bar{G}_{\sigma(t)}$ defined in Section 2 at time t , the relative estimation information $\sum_{j=0}^n a_{ij}(t) (\hat{y}_i - \hat{y}_j)$ is obtained using the communication infrastructure with $\hat{y}_i = C_{di} \hat{x}_i - \mathcal{C}_0 \hat{\chi}_{0i}$, $i = 1, 2, \dots, n$ and $\hat{y}_0 = 0$. In addition, $S(\epsilon) = \text{diag}(I_p \epsilon^{-1}, I_p \epsilon^{-2}, \dots, I_p \epsilon^{-n_0})$, where $\epsilon \in (0, 1]$ is a positive constant, and $\mathcal{P} = \mathcal{P}^T$ is a positive definite matrix satisfying

$$\mathcal{A}_0 \mathcal{P} + \mathcal{P} \mathcal{A}_0^T - 2\theta \mathcal{P} \mathcal{C}_0^T \mathcal{C}_0 \mathcal{P} + I_{pn_0} = 0, \quad (21)$$

and θ is a positive constant satisfying $\theta < \frac{1}{2} \min_{\bar{G}_k \in \bar{\mathbb{G}}_c} \min \Re\{\lambda(L_k)\}$.

Step IV: controller design

The observer-based controller is proposed as

$$u_i = F_i \hat{x}_i + (\Gamma_{1i} - F_i \Pi_{1i}) \hat{\omega}_i + (\Gamma_{2i} - F_i \Pi_{2i}) \hat{x}_{0i}, \quad (22)$$

where Π_{1i} , Γ_{1i} , Π_{2i} , and Γ_{2i} are the solutions of the following regulator equations

$$\Pi_{1i} S_i = A_i \Pi_{1i} + B_i \Gamma_{1i}, \quad (23a)$$

$$0 = D_{si} \Pi_{1i} + D_{wi}, \quad (23b)$$

$$\Pi_{2i} A_0 = A_i \Pi_{2i} + B_i \Gamma_{2i}, \quad (23c)$$

$$0 = D_{si} \Pi_{2i} + D_0, \quad i = 1, 2, \dots, n, \quad (23d)$$

and F_i is chosen such that $A_i + B_i F_i$ is Hurwitz.

Corollary 2 Let Assumptions 1, 2, 3 (the first item is replaced by that (A_i, C_{si}) is observable), and 4 hold and assume that $\kappa \geq \frac{\alpha + 4\theta \lambda_{\max}^2(\mathcal{P})}{1 - \alpha}$, where $\alpha \in (0, 1)$, θ and \mathcal{P} are given by (21). Also, let \hat{x}_i and $\hat{\omega}_i$ be obtained in (18), and \hat{x}_{0i} be obtained in (20). Then, there exists $\varepsilon_1^* \in (0, 1]$ such that, if $\varepsilon \in (0, \varepsilon_1^*]$, (22) ensures that $\lim_{t \rightarrow \infty} e_i(t) = 0$, $i = 1, 2, \dots, n$, for the multi-agent system (1)-(4).

Proof: The proof is straightforward following the similar analysis given in Lemmas 1 and 3. ■

5.2 Case II: (A_i, C_{di}) is observable

In this section, we use y_{si} to estimate ω_i and use ζ_i to estimate both x_i and x_0 . The control of each agent is supposed to have the structure shown in Fig. 4.

We also replace the first item of Assumption 3 with that (A_i, C_{di}) , for all $i = 1, 2, \dots, n$ is observable.

Step I: redundant mode remove

We first write the state and output of x_i and x_0 for each agent in the compact form

$$\begin{bmatrix} \dot{x}_i \\ \dot{x}_0 \end{bmatrix} = \begin{bmatrix} A_i & 0 \\ 0 & A_0 \end{bmatrix} \begin{bmatrix} x_i \\ x_0 \end{bmatrix} + \begin{bmatrix} B_i \\ 0 \end{bmatrix} u_i, \\ e_{di} = y_{di} - y_{d0} = \begin{bmatrix} C_{di} & -C_0 \end{bmatrix} \begin{bmatrix} x_i \\ x_0 \end{bmatrix}.$$

We can then construct a new state $\bar{x}_i = W_i \begin{bmatrix} x_i \\ x_0 \end{bmatrix}$ and perform

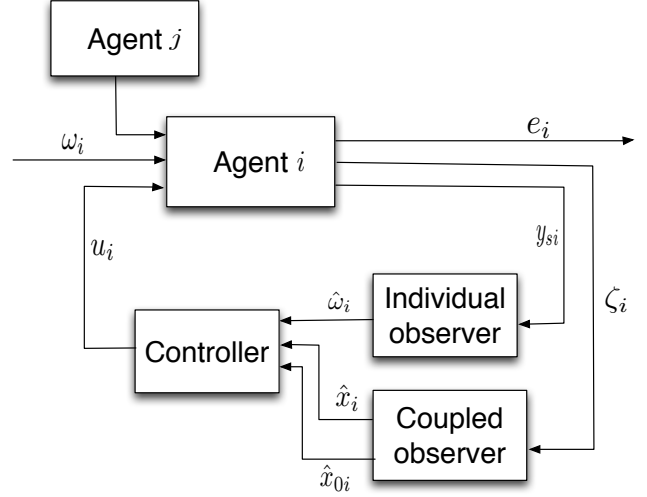


Fig. 4. Control architecture for agent v_i

the state transformation such that

$$\begin{aligned} \bar{x}_i &= \bar{A}_i \bar{x}_i + \bar{B}_i u_i = \begin{bmatrix} A_i & \bar{A}_{i12} \\ 0 & \bar{A}_{i22} \end{bmatrix} \bar{x}_i + \begin{bmatrix} B_i \\ 0 \end{bmatrix} u_i, \\ e_{di} &= \bar{C}_i \bar{x}_i = \begin{bmatrix} C_{di} & \bar{C}_{i12} \end{bmatrix} \bar{x}_i. \end{aligned}$$

Similarly, we can show that pair (\bar{A}_i, \bar{C}_i) is observable and the eigenvalues of \bar{A}_{i22} are a subset of the eigenvalues of A_0 , $i = 1, 2, \dots, n$.

Step II: coupled observer

We next define $\chi_i = T_i \bar{x}_i \in \mathbb{R}^{p\bar{n}}$, $i = 1, 2, \dots, n$, where $\bar{n} = n_0 + \max_{i=1,2,\dots,n} n_i$, and

$$T_i = \begin{bmatrix} \bar{C}_i \\ \vdots \\ \bar{C}_i \bar{A}_i^{\bar{n}-1} \end{bmatrix}.$$

Therefore, it follows that

$$\dot{\chi}_i = (\mathcal{A} + \mathcal{L}_i) \chi_i + \mathcal{B}_i u_i, \quad (26a)$$

$$e_{di} = \mathcal{C} \chi_i, \quad i = 1, 2, \dots, n, \quad (26b)$$

where $\mathcal{A} = \begin{bmatrix} 0 & I_{p(\bar{n}-1)} \\ 0 & 0 \end{bmatrix} \in \mathbb{R}^{p\bar{n} \times p\bar{n}}$, $\mathcal{L}_i = \begin{bmatrix} 0 \\ L_i \end{bmatrix}$, $\mathcal{B}_i = T_i \bar{B}_i$, $\mathcal{C} = \begin{bmatrix} I_p & 0 \end{bmatrix} \in \mathbb{R}^{p \times p\bar{n}}$ for some matrix $L_i \in \mathbb{R}^{p \times p\bar{n}}$.

Based on the neighbor-based group output information ζ_i , the distributed observer is proposed for (26) as

$$\dot{\hat{\chi}}_i = (\mathcal{A} + \mathcal{L}_i) \hat{\chi}_i + \mathcal{B}_i u_i + S(\varepsilon) \mathcal{P} \mathcal{C}^T$$

$$\times \left(\sum_{j=0}^n a_{ij}(t)(y_{di} - y_{dj}) - \sum_{j=0}^n a_{ij}(t)(\hat{y}_i - \hat{y}_j) \right), \quad (27a)$$

$$[\hat{x}_i^T, \hat{x}_{0i}^T]^T = W_i^{-1}(T_i^T T_i)^{-1} T_i^T \hat{\chi}_i, \quad i = 1, 2, \dots, n, \quad (27b)$$

where $a_{ij}(t)$, $i = 0, 1, \dots, n$, $j = 0, 1, \dots, n$, is entry (i, j) of the adjacency matrix $\bar{A}_{\sigma(t)}$ associated with $\bar{G}_{\sigma(t)}$ defined in Section 2 at time t , $\hat{y}_i = \mathcal{C} \hat{\chi}_i$, $i = 1, 2, \dots, n$, $\hat{y}_0 = 0$. In addition, $S(\varepsilon) = \text{diag}(I_p \varepsilon^{-1}, I_p \varepsilon^{-2}, \dots, I_p \varepsilon^{-n})$, where $\varepsilon \in (0, 1]$ is a positive constant, and $\mathcal{P} = \mathcal{P}^T$ is a positive definite matrix satisfying

$$\mathcal{A} \mathcal{P} + \mathcal{P} \mathcal{A}^T - 2\theta \mathcal{P} \mathcal{C}^T \mathcal{C} \mathcal{P} + I_{p\bar{n}} = 0, \quad (28)$$

where θ is a positive constant satisfying $\theta < \frac{1}{2} \min_{\bar{G}_k \in \bar{\mathcal{G}}_c} \min \Re\{\lambda(L_k)\}$.

Step III: individual observer

Based on the information of \hat{x}_i and the individual output information y_{si} , the following individual observer for each agent is proposed

$$\dot{\hat{\omega}}_i = S_i \hat{\omega}_i + K_{si} (C_{si} \hat{x}_i + C_{wi} \hat{\omega}_i - y_{si}), \quad i = 1, 2, \dots, n, \quad (29)$$

where K_{si} is chosen such that $S_i + K_{si} C_{wi}$ is Hurwitz stable.

Step IV: controller design

The observer-based controller is proposed as

$$u_i = F_i \hat{x}_i + (\Gamma_{1i} - F_i \Pi_{1i}) \hat{\omega}_i + (\Gamma_{2i} - F_i \Pi_{2i}) \hat{x}_{0i}, \quad (30)$$

where Π_{1i} , Γ_{1i} , Π_{2i} , and Γ_{2i} are the solutions of the following regulator equations

$$\Pi_{1i} S_i = A_i \Pi_{1i} + B_i \Gamma_{1i}, \quad (31a)$$

$$0 = D_{si} \Pi_{1i} + D_{wi}, \quad (31b)$$

$$\Pi_{2i} A_0 = A_i \Pi_{2i} + B_i \Gamma_{2i}, \quad (31c)$$

$$0 = D_{si} \Pi_{2i} + D_0, \quad i = 1, 2, \dots, n. \quad (31d)$$

and F_i is chosen such that $A_i + B_i F_i$ is Hurwitz.

Corollary 3 *Let Assumptions 1, 2, 3 (the first item is replaced by that (A_i, C_{di}) is observable), and 4 hold and assume that $\kappa \geq \frac{\alpha + 4\theta \lambda_{\max}^2(\mathcal{P})}{1 - \alpha}$, where $\alpha \in (0, 1)$, θ and \mathcal{P} are given by (28). Also, let \hat{x}_i and \hat{x}_{0i} be obtained in (27), and $\hat{\omega}_i$ be obtained in (29). Then, there exists $\varepsilon_2^* \in (0, 1]$ such that, if $\varepsilon \in (0, \varepsilon_2^*]$, (30) ensures that $\lim_{t \rightarrow \infty} e_i(t) = 0$, $i = 1, 2, \dots, n$, for the multi-agent system (1)-(4).*

Proof: See [1]. ■

6 Simulation Results

In this section, we illustrate the theoretical results. Consider a network of three agents as shown in Fig. 2. We assume that the adjacency matrix $\bar{A}_{\sigma(t)}$ associated with $\bar{G}_{\sigma(t)}$ is switching periodically. Denote $\ell = 0, 20, 40, \dots$

$$\bar{A} = \begin{cases} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, & \text{when } t \in [\ell, \ell + 6), \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{bmatrix}, & \text{when } t \in [\ell + 6, \ell + 12), \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}, & \text{when } t \in [\ell + 12, \ell + 18), \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}, & \text{when } t \in [\ell + 18, \ell + 20). \end{cases}$$

Example 1

We give an example to validate Theorem 1, the dynamics of

$$\text{the agents are described as } A_1 = \begin{bmatrix} 0 & 3 & 0 \\ 0 & 0 & 2 \\ 0 & -1 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

$$C_{s1} = C_{d1} = D_{s1} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}, A_2 = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, C_{s2} =$$

$$\begin{bmatrix} 1 & 0 \end{bmatrix}, C_{d2} = \begin{bmatrix} 0 & 1 \end{bmatrix}, D_{s2} = \begin{bmatrix} 1 & 1 \end{bmatrix}, A_3 = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix}, B_3 =$$

$$\begin{bmatrix} 0 \\ 1 \end{bmatrix}, C_{s3} = C_{d3} = D_{s3} = \begin{bmatrix} 1 & 0 \end{bmatrix}. \text{ The dynamics of the in-}$$

dividual autonomous exosystems are described as $S_i = 0$, $C_{wi} = D_{wi} = -1$, $i = 1, 2, 3$, and $\omega_1(0) = -2$, $\omega_2(0) = -4$, and $\omega_3(0) = -6$. The dynamics of the group autonomous

$$\text{exosystem are described as } A_0 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, C_0 = \begin{bmatrix} 1 & 0 \end{bmatrix},$$

$$D_0 = -C_0.$$

Following the design scheme proposed in Section 4, for the solutions of regulator equations (7), we have that

$$F_1 = \begin{bmatrix} -1 & -4.5 & -6 \end{bmatrix}, \quad \Pi_1 = \begin{bmatrix} 1 & 1.0345 & -0.4138 \\ 0 & 0.1379 & 0.3448 \\ 0 & -0.1724 & 0.0690 \end{bmatrix},$$

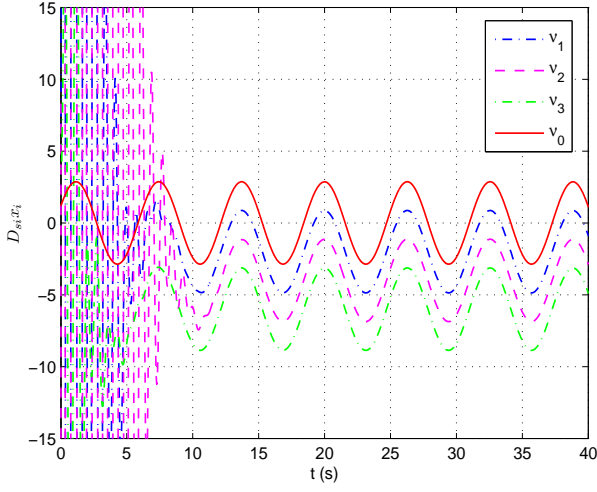


Fig. 5. Output convergence of system (1), (2), and (3) under the observer-based controller (15) for Theorem 1

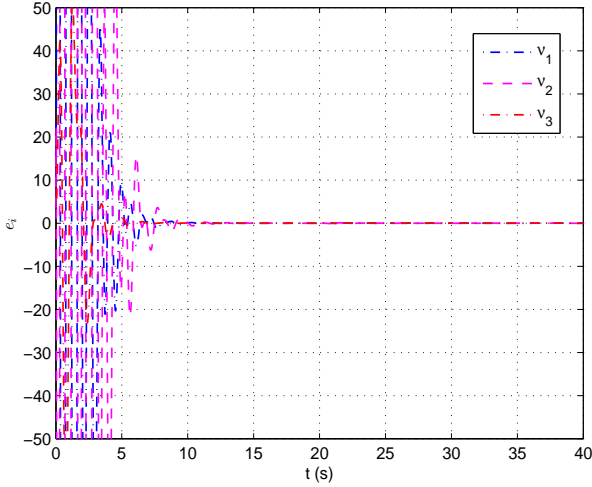


Fig. 6. Error convergence of system (1), (2), and (3) under the observer-based controller (15) for Theorem 1

$$\begin{aligned} \Gamma_1 &= \begin{bmatrix} 0 & 0.0690 & 0.1724 \end{bmatrix} \text{ for agent } v_1, F_2 = \begin{bmatrix} -2 & -6 \end{bmatrix}, \\ \Pi_2 &= \begin{bmatrix} 0 & 0.4 & -0.2 \\ 1 & 0.6 & 0.2 \end{bmatrix}, \Gamma_2 = \begin{bmatrix} 0 & -0.2 & 0.6 \end{bmatrix} \text{ for agent } v_2, \\ F_3 &= \begin{bmatrix} 0 & -1 \end{bmatrix}, \Pi_3 = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \Gamma_3 = \begin{bmatrix} 2 & 1 & 2 \end{bmatrix} \text{ for agent } v_3. \end{aligned}$$

We also have $\varepsilon = 0.2$ for (9) and $\theta = 0.1$ for (10).

Figs. 5 and 6 show, respectively, the state convergence and the error convergence of system (1), (2), and (3) under the observer-based controller (15). We see that coordinated output regulation is realized even when there exists multiple heterogeneous dynamics and the information interactions are switching. This agrees with Theorem 1.

Example 2

We next give an example to validate Corollary 2. In this section, the dynamics of the agents are described as

$$\begin{aligned} A_1 &= \begin{bmatrix} 0 & 3 & 0 \\ 0 & 0 & 2 \\ 0 & -1 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, C_{s1} = C_{d1} = D_{s1} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C_{s2} = C_{d2} = D_{s2} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \\ A_3 &= \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix}, B_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C_{s3} = C_{d3} = D_{s3} = \begin{bmatrix} 1 & 0 \end{bmatrix}. \end{aligned}$$

The dynamics of the individual autonomous exosystem are described as $\omega_1(t) = 0$, $\omega_2(t) = 0$, and $\omega_3(t) = 0$. The dynamics of the group autonomous exosystem are described

$$\text{as } A_0 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, C_0 = \begin{bmatrix} 1 & 0 \end{bmatrix}, D_0 = -C_0.$$

Following the design scheme proposed in Section 5.1, for the solutions of regulator equations (23), we have

$$\begin{aligned} \text{that } F_1 &= \begin{bmatrix} -1 & -4.5 & -6 \end{bmatrix}, \Pi_{21} = \begin{bmatrix} 1.0345 & -0.4138 \\ 0.1379 & 0.3448 \\ -0.1724 & 0.0690 \end{bmatrix}, \\ \Gamma_{21} &= \begin{bmatrix} 0.0690 & 0.1724 \end{bmatrix} \text{ for agent } v_1, F_2 = \begin{bmatrix} -2 & -3 \end{bmatrix}, \\ \Pi_{22} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \Gamma_{22} = \begin{bmatrix} -1 & 0 \end{bmatrix} \text{ for agent } v_2, F_3 = \begin{bmatrix} 0 & -1 \end{bmatrix}, \\ \Pi_{23} &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \Gamma_{23} = \begin{bmatrix} 1 & 2 \end{bmatrix} \text{ for agent } v_3. \end{aligned}$$

We also have $K_{a1} = [-0.75, -4, -1.25]^T$, $K_{a2} = [-3, -2]^T$, $K_{a3} = [-1, 2]^T$ for (18), $\varepsilon = 0.2$ for (20) and $\theta = 0.1$ for (21).

Figs. 7 and 8 show, respectively, the state convergence and the error convergence of system (1), (2), and (3) under the observer-based controller (22). We see that coordinated output regulation is realized even when there exists multiple heterogeneous dynamics and the information interactions are switching. This agrees with Corollary 2.

Example 3

We give an example to validate Corollary 3, the dynamics of the agents are described as

$$\begin{aligned} A_1 &= \begin{bmatrix} 0 & 3 & 0 \\ 0 & 0 & 2 \\ 0 & -1 & 0 \end{bmatrix}, B_1 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, C_{s1} = C_{d1} = D_{s1} = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, B_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \end{aligned}$$

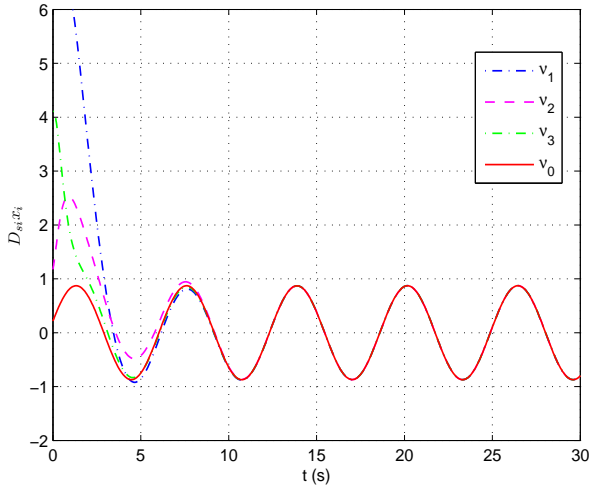


Fig. 7. Output convergence of system (1), (2), and (3) under the observer-based controller (22) for Corollary 2

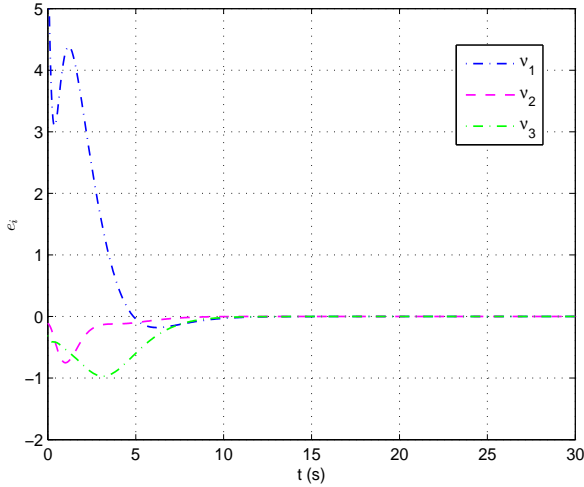


Fig. 8. Error convergence of system (1), (2), and (3) under the observer-based controller (22) for Corollary 2

$\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C_{s2} = C_{d2} = D_{s2} = \begin{bmatrix} 1 & 0 \end{bmatrix}$. $A_3 = \begin{bmatrix} 0 & 1 \\ -2 & -2 \end{bmatrix}$, $B_3 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C_{s3} = C_{d3} = D_{s3} = \begin{bmatrix} 1 & 0 \end{bmatrix}$. The dynamics of the individual autonomous exosystems are described as $S_i = 0$, $C_{wi} = D_{wi} = -1$, $i = 1, 2, 3$, and $\omega_1(0) = -2$, $\omega_2(0) = -4$, and $\omega_3(0) = -6$. The dynamics of the group autonomous exosystem are described as $A_0 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$, $C_0 = \begin{bmatrix} 1 & 0 \end{bmatrix}$, $D_0 = -C_0$.

Following the design scheme proposed in Section 5.2, for the solutions of regulator equations (31), we have

that $F_1 = \begin{bmatrix} -1 & -4.5 & -6 \end{bmatrix}$, $\Pi_{11} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\Gamma_{11} = 0$, $\Pi_{21} = \begin{bmatrix} 1.0345 & -0.4138 \\ 0.1379 & 0.3448 \\ -0.1724 & 0.0690 \end{bmatrix}$, $\Gamma_{21} = \begin{bmatrix} 0.0690 & 0.1724 \end{bmatrix}$ for agent v_1 , $F_2 = \begin{bmatrix} -2 & -3 \end{bmatrix}$, $\Pi_{12} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\Gamma_{12} = 0$, $\Pi_{22} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\Gamma_{22} = \begin{bmatrix} -1 & 0 \end{bmatrix}$ for agent v_2 , $F_3 = \begin{bmatrix} 0 & -1 \end{bmatrix}$, $\Pi_{13} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $\Gamma_{13} = -2$, $\Pi_{23} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $\Gamma_{23} = \begin{bmatrix} 1 & 2 \end{bmatrix}$ for agent v_3 . We also have $\varepsilon = 0.2$ for (27), $\theta = 0.1$ for (28), and $K_{si} = 1$, $i = 1, 2, 3$ for (29),

Figs. 9 and 10 show, respectively, the state convergence and the error convergence of system (1), (2), and (3) under the observer-based controller (30). We see that coordinated output regulation is realized even when there exists multiple heterogeneous dynamics and the information interactions are switching. This agrees with Corollary 3.

7 Conclusions

This paper studied the coordinated output regulation problem of multiple heterogeneous linear systems. We first formulated the coordinated output regulation problem and specified the information that is available for each agent. A high-gain based distributed observer and an individual observer were introduced for each agent and observer-based controllers were designed to solve the problem. The information interactions among the agents and the group autonomous exosystem were allowed to be switching over a finite set of fixed networks containing both the graph having a spanning tree and the graph having not. The relationship of the information interactions, the dwell time, the non-identical dynamics of different agents, and the high-gain parameters were also given. Simulations were given to validate the theoretical results. Future directions include relaxing the dwell-time assumption.

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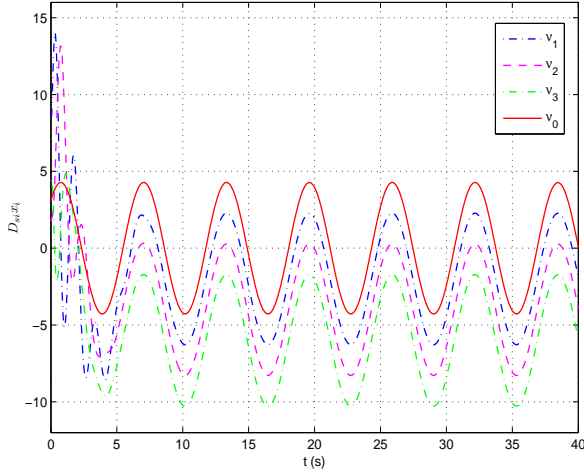


Fig. 9. Output convergence of system (1), (2), and (3) under the observer-based controller (30) for Corollary 3

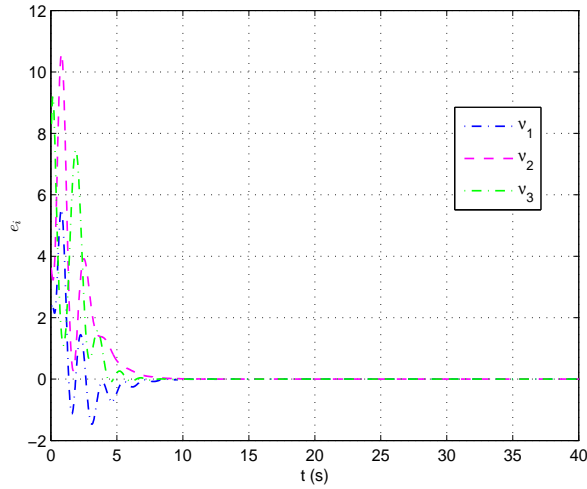


Fig. 10. Error convergence of system (1), (2), and (3) under the observer-based controller (30) for Corollary 3

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